

The questions below correspond to the material of chapter of Introduction to Analysis by Rosenlicht. If you are not familiar with this material, please read this chapter, and come to office hours to discuss it. I encourage group work as well, but it is important to write up your own solutions.

1. The first question is just to get to know you better.
  - (a) What are you most excited for in this class?
  - (b) What made you decide to pursue a graduate degree in math?
  - (c) What is your favorite thing about math?
2. Let  $X$  be a set and  $\{A_i\}_{i \in \mathbb{N}}$  be a collection of subsets of  $X$ . Show that

$$X - (\cap_{i \in \mathbb{N}} A_i) = \cup_{i \in \mathbb{N}} (X - A_i)$$

3. Let  $X$  be a set and  $\{A_i\}_{i \in \mathbb{N}}$  be a collection of subsets of  $X$ . Show that

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4. Given sets  $A, B_1$  and  $B_2$  show that

$$A \times (B_1 \cup B_2) = (A \times B_1) \cup (A \times B_2)$$

5. Let  $f : S \rightarrow T$  be a function from  $S$  to  $T$ . If  $A$  and  $B$  are subsets of  $S$  show that

$$f(A \cap B) \subset f(A) \cap f(B)$$

6. Use induction to show that  $m < 2^m$  for any positive integer  $n$ .
7. For any positive integer  $n$  and  $x \in \mathbb{R}$  show that

$$\sum_{i=0}^n x^i = \frac{1 - x^{n+1}}{1 - x}$$

8. Let  $f : S \rightarrow T$  be a function from  $S$  to  $T$ . If  $A$  and  $B$  are subsets of  $T$  show that

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$

9. Let  $f : S \rightarrow T$  be a function from  $S$  to  $T$ . If  $A$  and  $B$  are subsets of  $T$  show that

$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$$

10. Let  $n$  and  $m$  be positive integers. Let  $N$  be a set with  $n$  elements and  $M$  a set with  $m$  elements. How many functions are there from  $N$  to  $M$ ? How many injective functions? How many surjective functions?
11. Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $g \circ f : X \rightarrow Z$  is injective show that  $f$  is injective. If  $g \circ f : X \rightarrow Z$  is surjective show that  $g$  is surjective.